Effective action in general chiral superfield model

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According to the superstring theory the low-energy elementary particle models contain as ingredient the multiplets of chiral and antichiral superfields action of which is given in terms of kählerian effective potential $K(\bar{\Phi}, \Phi)$ and chiral $W(\Phi)$ and antichiral $\bar{W}(\bar{\Phi})$ potentials. These potentials are found in explicit and closed form within string perturbation theory (see f.e. [1]). Phenomenological aspects of such models have been studied in recent papers [2]. In quantum theory one can expect an appearance of quantum corrections to the potentials $K(\bar{\Phi}, \Phi)$ and $W(\Phi)$. As a result we face a problem of calculating effective action in models with arbitrary functions $K(\bar{\Phi}, \Phi)$ and $W(\Phi)$.

The remarkable features of the massless theories with $N = 1$ chiral superfields are the possibilities of obtaining the chiral quantum corrections. A few years ago West [3] pointed out that finite two-loop chiral contribution to effective action really arises in massless Wess-Zumino model (see also [4]).

In this talk we consider the general problem of calculating leading quantum correction to chiral potential and kählerian potential in theory with arbitrary potentials $K(\bar{\Phi}, \Phi)$ and $W(\Phi)$, $\bar{W}(\bar{\Phi})$. The remarkable result we obtain here is that despite the theory under consideration is non-renormalizable at arbitrary $K(\bar{\Phi}, \Phi)$, $W(\Phi)$, the lower (two-loop) chiral correction to effective action is always finite.

We consider $N = 1$ supersymmetric field theory with action

$$S[\bar{\Phi}, \Phi] = \int d^8 z K(\bar{\Phi}, \Phi) + (\int d^6 z W(\Phi) + h.c.) \tag{1}$$

where $\Phi(z)$ and $\bar{\Phi}(z)$ are chiral and antichiral superfields respectively. As well known, the real function $K(\bar{\Phi}, \Phi)$ is called kählerian potential and holomorphic function $W(\Phi)$ is called chiral potential [5]. The partial cases of the theory (1) are Wess-Zumino model with $K(\bar{\Phi}, \Phi) = \Phi\bar{\Phi}$, $W(\Phi) \sim \Phi^3$ and $N = 1$ supersymmetric four-dimensional sigma-model with $W(\Phi) = 0$. The action (1) is a most general one constructed from chiral and antichiral superfields which does not contain the higher derivatives at component level. Therefore it is natural to call the theory (1) a general chiral superfield model.

Let $\Gamma[\bar{\Phi}, \Phi]$ be effective action in the model (1). Within a momentum expansion the effective action can be presented as a series in supercovariant

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derivatives $D_A = (\partial_a, D_\alpha, \bar{D}_{\dot\alpha})$ in the form

$$
\begin{aligned}
\Gamma[\bar{\Phi}, \Phi] &= \int d^8 z \mathcal{L}_{eff}(\Phi, D_A\Phi, D_A D_B\Phi; \bar{\Phi}, D_A\bar{\Phi}, D_A D_B\bar{\Phi}) + \\
&+ (\int d^6 z \mathcal{L}_{eff}^{(c)}(\Phi) + h.c.) + \ldots
\end{aligned} \tag{2}
$$

Here $\mathcal{L}_{eff}$ is called general effective lagrangian, $\mathcal{L}_{eff}^{(c)}$ is called chiral effective lagrangian. Both these lagrangians are the series in supercovariant derivatives of superfields and can be written as follows

$$
\begin{aligned}
\mathcal{L}_{eff} &= K_{eff}(\bar{\Phi}, \Phi) + \ldots = K(\bar{\Phi}, \Phi) + \sum_{n=1}^{\infty} K_{eff}^{(n)}(\bar{\Phi}, \Phi) \\
\mathcal{L}_{eff}^{(c)} &= W_{eff}(\Phi) + \ldots = W(\Phi) + \sum_{n=1}^{\infty} W_{eff}^{(n)}(\Phi) + \ldots
\end{aligned} \tag{3}
$$

Here dots mean terms depending on covariant derivatives of superfields $\Phi, \bar{\Phi}$. Here $K_{eff}(\bar{\Phi}, \Phi)$ is called kahlerian effective potential, $W_{eff}(\Phi)$ is called chiral (or holomorphic) effective potential, $K_{eff}^{(n)}$ is a n-th correction to kahlerian potential and $W_{eff}^{(n)}$ is a n-th correcton to chiral (holomorphic) potential W.

To consider the effective lagrangians $\mathcal{L}_{eff}$ and $\mathcal{L}_{eff}^{(c)}$ we use path integral representation of the effective action [5, 6]

$$
\begin{aligned}
\exp(\frac{i}{\hbar}\Gamma[\bar{\Phi}, \Phi]) &= \int \mathcal{D}\phi \mathcal{D}\bar{\phi} \exp(\frac{i}{\hbar}S[\bar{\Phi} + \sqrt{\hbar}\bar{\phi}, \Phi + \sqrt{\hbar}\phi] - \\
&- (\int d^6 z \frac{\delta\Gamma[\bar{\Phi}, \Phi]}{\delta\Phi(z)}\phi(z) + h.c.))
\end{aligned} \tag{4}
$$

Here $\Phi, \bar{\Phi}$ are the background superfields and $\phi, \bar{\phi}$ are the quantum ones. The effective action can be written as $\Gamma[\bar{\Phi}, \Phi] = S[\bar{\Phi}, \Phi] + \tilde{\Gamma}[\bar{\Phi}, \Phi]$, where $\tilde{\Gamma}[\bar{\Phi}, \Phi]$ is a quantum correction. Eq. (4) allows to obtain $\tilde{\Gamma}[\bar{\Phi}, \Phi]$ in form of loop expansion $\tilde{\Gamma}[\bar{\Phi}, \Phi] = \sum_{n=1}^{\infty} \hbar^n \Gamma^{(n)}[\bar{\Phi}, \Phi]$ and hence, to get loop expansion for the effective lagrangians $\mathcal{L}_{eff}$ and $\mathcal{L}_{eff}^{(c)}$.

To find loop corrections $\Gamma^{(n)}[\bar{\Phi}, \Phi]$ in explicit form we expand right-hand side of eq. (4) in power series in quantum superfields $\phi, \bar{\phi}$. As usual, the quadratic part of expansion of $\frac{1}{\hbar}S[\bar{\Phi} + \sqrt{\hbar}\bar{\phi}, \Phi + \sqrt{\hbar}\phi]$

$$
S_2 = \frac{1}{2}\int d^8 z \begin{pmatrix} \phi & \bar{\phi} \end{pmatrix} \begin{pmatrix} K_{\Phi\Phi} & K_{\Phi\bar{\Phi}} \\ K_{\Phi\bar{\Phi}} & K_{\bar{\Phi}\bar{\Phi}} \end{pmatrix} \begin{pmatrix} \phi \\ \bar{\phi} \end{pmatrix} + [\int d^6 z \frac{1}{2}W''\phi^2 + h.c.] \tag{5}
$$

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defines the propagators and the higher terms of expansion define the vertices. Here $K_{\Phi\bar\Phi} = \frac{\partial^2 K(\bar\Phi,\Phi)}{\partial\Phi\partial\bar\Phi}$, $K_{\Phi\Phi} = \frac{\partial^2 K(\bar\Phi,\Phi)}{\partial\Phi^2}$ etc, $W'' = \frac{d^2 W}{d\Phi^2}$.

Let us consider holomorphic effective potential $W_{eff}(\Phi)$. It was noted by West [3] that non-renormalization theorem (see f.e. [5]) does not forbid an existence of the finite chiral corrections to the effective action. The matter is the theories with chiral superfields admit the loop corrections of the form $\int d^8 z f(\Phi)(-\frac{D^2}{4\Box})g(\Phi) = \int d^6 z f(\Phi)g(\Phi)$ where $f(\Phi), g(\Phi)$ are some functions of chiral superfield Φ. We note that this equation shows the superfield Φ is not a constant. It is easy to prove that the chiral contributions to effective action can be generated by supergraphs containing massless propagators only. To find chiral corrections to effective action we put $\bar\Phi = 0$ in eqs. (4,5). **Therefore here and further all derivatives of K, W and $\bar W$ will be taken at $\bar\Phi = 0$.** Under this condition the action of quantum superfields $\phi, \bar\phi$ in external superfield Φ looks like

$$S[\bar\phi, \phi, \Phi] = \frac{1}{2}\int d^8 z \begin{pmatrix} \phi & \bar\phi \end{pmatrix} \begin{pmatrix} K_{\Phi\Phi} & K_{\Phi\bar\Phi} \\ K_{\Phi\bar\Phi} & K_{\bar\Phi\bar\Phi} \end{pmatrix} \begin{pmatrix} \phi \\ \bar\phi \end{pmatrix} + \int d^6 z \frac{1}{2} W'' \phi^2 + \dots \quad (6)$$

The dots here denote the terms of third, fourth and higher orders in quantum superfields. We call the theory massless if $W''|_{\Phi=0} = 0$. Further we consider only massless theory.

To calculate the corrections to $W(\Phi)$ we use supergraph technique (see f.e. [5]). For this purpose one splits the action (6) into sum of free part and vertices of interaction. As a free part we take the action $S_0 = \int d^8 z \phi\bar\phi$. The corresponding superpropagator is $G(z_1, z_2) = -\frac{D_1^2 \bar D_2^2}{16\Box}\delta^8(z_1 - z_2)$. And the term $S[\bar\phi, \phi, \Phi] - S_0$ will be treated as vertices where $S[\bar\phi, \phi, \Phi]$ is given by eq. (6). Our purpose is to find first leading contribution to $W_{eff}(\Phi)$. As we will show, chiral loop contributions are began with two loops. Therefore we keep in eq. (6) only the terms of second, third and fourth orders in quantum fields.

Non-trivial corrections to chiral potential can arise only if $2L + 1 - n_{W''} - n_{V_c} = 0$ where L is a number of loops, $n_{W''}$ is a number of vertices proportional to W'', n_{V_c} is that one of vertices of third and higher orders in quantum superfields, otherwise corresponding contribution will either vanish or lead to singularity in infrared linit. In one-loop approximation this equation leads to $n_{W''} + n_{V_c} = 3$. However, all supergraphs satisfying this condition have zero contribution. Therefore first correction to chiral effective potential is two-loop one. In two-loop approximation this equation has the form $n_{W''} + n_{V_c} = 5$. Since the number of purely chiral (antichiral) vertices independent of W'' in two-loop supergraphs can be equal to 0, 1 or 2, number of external vertices W'' takes values from 3 to 5.

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We note that non-trivial contribution to holomorphic effective potential from any diagram can arise only if number of D^2-factors is more by one than the number of $\bar{D}^2$-factors (see details in [9]). The only Green function in the theory is a propagator $< \phi\bar{\phi} >$. Therefore total number of quantum chiral superfields ϕ corresponding to all vertices must be equal to that one of antichiral ones $\bar{\phi}$. As a result we find that the only two-loop supergraph contributing to chiral effective potential looks like
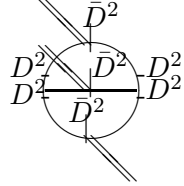
Here internal line is a propagator $< \phi\bar{\phi} >$ depending on background chiral superfields which has the form

$$< \phi\bar{\phi} > = -\bar{D}_1^2 D_2^2 \frac{\delta^8(z_1 - z_2)}{16 K_{\Phi\bar{\Phi}}(z_1)\square}. \tag{7}$$

We note that the superfield $K_{\Phi\bar{\Phi}}$ is not constant here. Double external lines are W''.

After D-algebra transformations and loop integrations we find that two-loop contribution to holomorphic effective potential in this model looks like

$$W^{(2)} = \frac{6}{(16\pi^2)^2}\zeta(3)\bar{W}'''^2\left\{\frac{W''(z)}{K_{\Phi\bar{\Phi}}^2(z)}\right\}^3 \tag{8}$$

One reminds that $\bar{W}''' = \bar{W}'''(\bar{\Phi})|_{\bar{\Phi}=0}$ and $K_{\Phi\bar{\Phi}}(z) = \frac{\partial^2 K(\bar{\Phi},\Phi)}{\partial\Phi\partial\bar{\Phi}}|_{\bar{\Phi}=0}$ here. We see that the correction (8) is finite and does not require renormalization.

Now we turn to studying of quantum contributions to kählerian effective potential depending only on superfields Φ, $\bar{\Phi}$ but not on their derivatives.
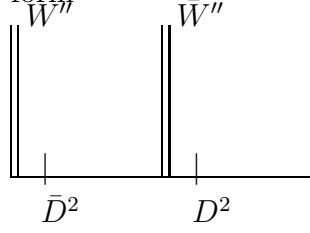
The one-loop diagrams contributing to kahlerian effective potential are
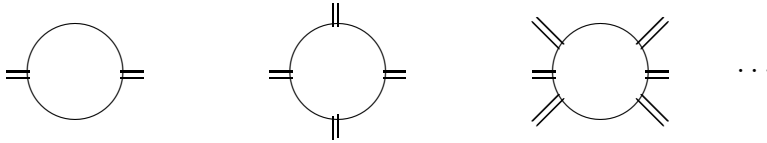
Double external lines correspond to alternating W'' and $\bar{W}''$. Internal lines are $< \phi\bar{\phi} >$-propagators of the form

$$G_0 \equiv < \phi\bar{\phi} > = -\frac{\bar{D}^2 D^2}{16 K_{\Phi\bar{\Phi}}\square}$$

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Supergraph of such structure with $2n$ legs represents itself as a ring containing of n links of the following form

The total contribution of all these diagrams after D-algebra transformations, summarizing, integration over momenta and subtraction of divergences is equal to
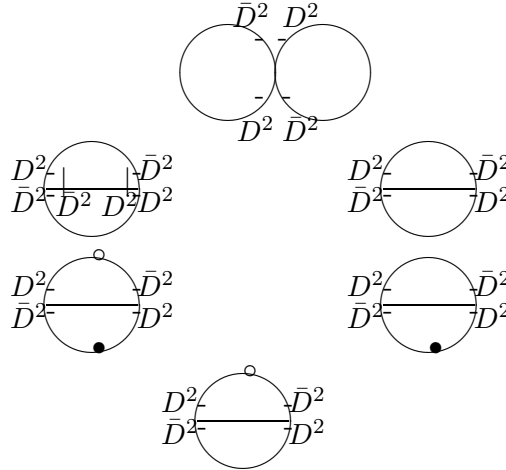
$$K^{(1)} = -\int d^4\theta \frac{1}{32\pi^2} \frac{W''\bar{W}''}{K^2_{\Phi\bar{\Phi}}} \ln\left(\frac{W''\bar{W}''}{\mu^2 K^2_{\Phi\bar{\Phi}}}\right) \qquad (9)$$

We can find also two-loop correction to kählerian potential. Since kählerian effective potential depends only on superfields $\Phi, \bar{\Phi}$ but not on their derivatives supergraphs contributing to it must include equal number of D^2 and $\bar{D}^2$ factors with all vertices rewritten in the form of an integral over whole superspace.

Components of matrix superpropagator for the case of constant superfields looks like

$$\begin{aligned}
G_{++} &= \frac{\bar{W}''}{K_{\Phi\bar{\Phi}}\Box + |W''|^2} \frac{\bar{D}_1^2}{4}\delta_{12}; \quad G_{+-} = \frac{1}{K_{\Phi\bar{\Phi}}\Box + |W''|^2} \frac{\bar{D}_1^2 D_2^2}{16}\delta_{12} \\
G_{-+} &= \frac{1}{K_{\Phi\bar{\Phi}}\Box + |W''|^2} \frac{D_1^2 \bar{D}_2^2}{16}\delta_{12}; \quad G_{--} = \frac{W''}{K_{\Phi\bar{\Phi}}\Box + |W''|^2} \frac{D_1^2}{4}\delta_{12} \quad (10)
\end{aligned}$$

The all diagrams with equal number of D^2 and $\bar{D}^2$-factors are given in this picture.

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Here

$$\text{———} = \frac{1}{K_{\Phi\bar\Phi}\Box + |W''|^2} \qquad \text{——}\circ\text{——} = \frac{W''}{K_{\Phi\bar\Phi}\Box + |W''|^2}$$

$$\text{——}\bullet\text{——} = \frac{\bar W''}{K_{\Phi\bar\Phi}\Box + |W''|^2}$$

Two-loop correction to kählerian potential is a sum of all these contributions. After D-algebra transformations, loop integrations and subtraction of one-loop and two-loop divergences it is equal to

$$
\begin{aligned}
K^{(2)} &= \frac{1}{(16\pi^2)^2}\Big(\frac{|W''|^2}{K_{\Phi\bar\Phi}{}^4}\big(-\frac{1}{4}\ln^2\frac{|W''|^2}{\mu^2 K_{\Phi\bar\Phi}} + \frac{3-\gamma}{2}\ln\frac{|W''|^2}{\mu^2 K_{\Phi\bar\Phi}} + \frac{3}{2}(\gamma-1) + \\
&+ \frac{1}{4}(\gamma^2 + \zeta(2)))\big)\Big\{\frac{1}{6}(K_{\Phi\bar\Phi}{}^3|W'''|^2 + (\bar W''W'''K_{\Phi\bar\Phi}{}^2 K_{\Phi\Phi\bar\Phi} + h.c.) + \\
&+ W''\bar W'' K_{\Phi\bar\Phi}K_{\Phi\Phi\bar\Phi}K_{\Phi\bar\Phi\bar\Phi} + \\
&+ K_{\Phi\bar\Phi}{}^2|W''|^2|K_{\Phi\Phi\bar\Phi}|^2) + \frac{1}{2}K_{\Phi\bar\Phi}{}^2|W''|^2 K_{\Phi\Phi\bar\Phi}K_{\Phi\bar\Phi\bar\Phi}\Big\} + \frac{1}{(16\pi^2)^2} \times \\
&\times \frac{|W''|^4}{K_{\Phi\bar\Phi}{}^4}(\gamma-1+\ln\frac{|W''|^2}{\mu^2 K_{\Phi\bar\Phi}})^2(K_{\Phi\bar\Phi}{}^2 K_{\Phi\Phi\bar\Phi}K_{\Phi\bar\Phi\bar\Phi} + K_{\Phi\bar\Phi}{}^2 K_{\Phi\Phi\bar\Phi\bar\Phi})
\end{aligned}
\tag{11}
$$

The value of normalization point μ can be fixed by imposing of suitable normalization condition.

To conclude, we have solved the problem of calculating leading holomorphic correction to superfield effective action in general chiral superfield model (1) with arbitrary potentials $K(\bar\Phi,\Phi)$ and $W(\Phi)$. The result has the universal form (8) and it is finite independently if the functions $K(\bar\Phi,\Phi)$, $W(\Phi)$ correspond to renormalizable theory or no. We have also calculated one-loop and two-loop contributions to kählerian effective potential. We note that results (8,9,11) reproduces the known results for Wess-Zumino model at $W = -\frac{\lambda}{3!}\Phi^3$, $K = \bar\Phi\Phi$ [3, 4].

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